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Application of the simplified fixed-scatterer approximation to elastic scattering of electrons and positrons by helium atoms

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Abstract : The elastic scattering of fast electrons and positrons by helium atoms is studied by employing the fixed scatterer approximation retaining up to double scattering terms. Differential as well as total cross sections are obtained both for direct and exchange cases for incident energies ranging from 50–3000 eV. Inclusion of exchange effects has improved the results in the forward direction. Our theoretical values for e-He differential and total cross sections are in excellent agreement with other theoretical and experimental data.

1. Introduction

The scattering of electrons by helium atoms is studied most by experimental physicists. Now that the behaviour of the cross-section for e^--He scattering processes is rather well known. The theoretical workers take this system as a testing ground of the validity of the methods. Apart from availability of measured values, this system is the easiest to handle after e^--H processes.

After the work of Callaway *et al* (1968) and Burke *et al* (1963) the low energy behaviour of e^--He scattering is rather settled. The recent trend is to find a suitable method for intermediate and high energies. After the prescription of Chase (1956), the approximate forms of the fixed scatterer model have been used by many workers to investigate the electron-atom collision. The Glauber and eikonal Glauber methods may be derived directly from the fixed scatterer approximation (FSA). Applications of Glauber model to invsetigate the e^--He scattering have been made by many workers. The eikonal-Born-series (EBS) method has been used by Byron and Joachain (1977). Saha *et al* (1973) have applied the modified eikonal method. Out of these methods, eikonal-Born series results are in best agreement with measured values.

Here we have used a simplified form of the FSA as proposed by Ghosh (1977) to investigate the e^--H problem. In this paper Ghosh has pointed out the basic difference between Glauber's and his own form. He has also discussed the difference between the present form of the FSA and the corresponding simplified form of the second Born approximation as proposed by Massey and Mohr (1934). Apart from the frozen target approximation, he has in the present model neglected the third and higher order scattering terms. The results obtained by Ghosh for elastic and inelastic cases in e^--H scattering are very encouraging.

In the present paper we have applied the method as proposed by Ghosh to investigate the e^--He elastic scattering in the energy region from 50 eV to 3000 eV.

2. Theory

We can write the Schrödinger equation for e^--He scattering in the frame work of FSA as

$$(E-E_B-T_1)\psi^+_{k_i}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3) = v(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3)\psi^+_{k_i}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3) \tag{1}$$

where the total wave function of the system, $\psi^+_{k_i}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3)$ is expressed as the product of the target wave function $\phi_1(\boldsymbol{r}_2, \boldsymbol{r}_3)$ and the incident wave $F^+_{\boldsymbol{k}_i}(\boldsymbol{r}_1)$ in the FSA, i.e

$$\psi_{\boldsymbol{k}_i}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3) = \Phi_1(\boldsymbol{r}_2, \boldsymbol{r}_3)F^+_{\boldsymbol{k}_i}(\boldsymbol{r}_1), \tag{2}$$

where k_i is the incident momentum and $v(r_1, r_2, r_3)$ is the perturbed potential. Here E_B is the binding energy of the target atom and T_1 the K.E. of the incident particle. Equation (1) takes the form

$$(E-E_B-T_1)F^+_{\boldsymbol{k}_i}(\boldsymbol{r}_1) = v(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3)F^+_{k_i}(\boldsymbol{r}_1). \tag{3}$$

Here the function $F^+_{\boldsymbol{k}_i}(\boldsymbol{r}_1)$ which has a parametric dependence on the target electrons, r_2, r_3, satisfies the usual boundary condition. The corresponding Lippman Schwinger equation is given by

$$F^+_{\boldsymbol{k}_i}(\boldsymbol{r}_1) = |\boldsymbol{k}_i> + \frac{2}{(2\pi)^3}\int \frac{d\boldsymbol{k}''|\boldsymbol{k}''><\boldsymbol{k}''|V(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3)|F^+_{\boldsymbol{k}_i}(\boldsymbol{r}_1)>}{k_i^2-k''^2+i\epsilon} \tag{4}$$

Equation (4) in the FSA is exact until now. $F_{\boldsymbol{k}_i}(r_1)$ on the right hand side of equation (4) is now replaced by a plane wave i.e. we retain upto the second order terms. Thus the function $F_{\boldsymbol{k}_i}(\boldsymbol{r}_1)$ take the form, with proper normalisation.

$$F^+_{\boldsymbol{k}_i}(\boldsymbol{r}_1) = |\boldsymbol{k}_i> + \frac{1}{4\pi^3}\int \frac{d\boldsymbol{k}''|\boldsymbol{k}''> <\boldsymbol{k}''|V|\boldsymbol{k}_i>}{k_i^2-k''^2-ic}$$

$$= e^{i\boldsymbol{k}_i.\boldsymbol{r}_1} + \frac{1}{\pi^2}\int \frac{d\boldsymbol{k}'' e^{i\boldsymbol{k}.\boldsymbol{r}_1}}{k_i^2-k''^2-i\epsilon}\left[\frac{2-e^{i(\boldsymbol{k}_i-\boldsymbol{k}'').\boldsymbol{r}_2}-e^{i(\boldsymbol{k}_i-\boldsymbol{k}'')\boldsymbol{r}_3}}{(\boldsymbol{k}_i-\boldsymbol{k}'')^2}\right]$$

The direct scattering amplitude from the initial state $|i>$ with momentum $\boldsymbol{k}_i$ to the final state $<f|$ with momentum $\boldsymbol{k}_f$ is given by

$$F^{FSA}_{fi}(k_f, k_i) = \frac{-\mu_f}{2\pi} < \Phi_f(\boldsymbol{r}_2, \boldsymbol{r}_3)\boldsymbol{k}_f | V | F^+_{k_i}(\boldsymbol{r}_1)\Phi_i(\boldsymbol{r}_2, \boldsymbol{r}_3) >$$

$$= f^s + T^d,$$

where

$$f^s = -\frac{1}{2\pi} < \Phi_f(\boldsymbol{r}_2, \boldsymbol{r}_3)| V | \Phi_i(\boldsymbol{r}_2, \boldsymbol{r}_3) >,$$

and

$$T^d = \frac{1}{8\pi^4}\int \Phi_f(\boldsymbol{r}_2, \boldsymbol{r}_3)\int \frac{d\boldsymbol{k}'' <\boldsymbol{k}_f|V|\boldsymbol{k}''> <\boldsymbol{k}''|V|\boldsymbol{k}_i>}{k''^2-k_i^2-i\epsilon}\Phi_i(\boldsymbol{r}_2, \boldsymbol{r}_3)d\boldsymbol{r}_2\, d\boldsymbol{r}_3.$$

The calculation of f^s, which is in fact the first Born term, is straightforward The final expression for T^d is lengthy and the type of integrals from which one can obtain the final expression for T^d are given in Appendix I

The corresponding rearrangement amplitude is

$$g^{FSA}_{fi} = \frac{-\mu_f}{2\pi} < \Phi_f(\boldsymbol{r}_2, \boldsymbol{r}_3)\boldsymbol{k}_f| V_{post}|\ \Phi_i(\boldsymbol{r}_2, \boldsymbol{r}_3)F^+_{\boldsymbol{k}_i} > = g^s + T^g$$

where g^s is the Oppenheimer amplitude and can easily be calculated. The double scattering term of the exchange amplitude T^g is given by

$$T^g = \frac{-1}{2\pi^3}\Phi_f(\boldsymbol{r}_2, \boldsymbol{r}_3)\ e^{-i\boldsymbol{k}_f.\boldsymbol{r}_2}\left(-\frac{2}{r_2}+\frac{1}{r_{12}}+\frac{1}{r_{23}}\right)\Phi_i\ (\boldsymbol{r}_2, \boldsymbol{r}_3)\ e^{i\boldsymbol{k}''.\boldsymbol{r}_1}$$

$$\times\frac{2-e^{i(\boldsymbol{k}_i-\boldsymbol{k}'').\boldsymbol{r}_2}-e^{i(\boldsymbol{k}_i-\boldsymbol{k}'').\boldsymbol{r}_3}}{(k''^2-k_i^2-i\epsilon)(\boldsymbol{k}_i-\boldsymbol{k}'')^2}\ d\boldsymbol{k}\, d\boldsymbol{r}_1\, d\boldsymbol{r}_2\, d\boldsymbol{r}_3$$

The final expression for T^g is also very lengthy and the type integrals from which the final expression can be obtained are given in Appendix II.

The wave function used for the ground state of helium atom is (Mott and Massey 1965)

$$\Phi(\mathbf{r}_2, \mathbf{r}_3) = \frac{N^2}{\pi}\left[e^{-Z_1 r_2} + ce^{-2Z_1 r_2}\right] \times \left[e^{-Z_1 r_3} + ce^{-2Z_1 r_3}\right]$$

where $N = 1{\cdot}484$, $Z_1 = 1{\cdot}456$, $c = 0{\cdot}6$ and V is of the form

$$V = -\frac{2}{r_1} + \frac{1}{r_{12}} + \frac{1}{r_{13}}$$

3. Results and discussion

We present here the results for the elastic scattering of electrons and positrons by helium atoms in the fixed scatterer approximation for incident energies ranging from 50 eV to 3000 eV. In the case of positron scattering we have simply to change the sign of the first Born approximation (FBA) scattering amplitude of electrons. The integrals in the scattering amplitude have been evaluated numerically by the Gauss-Legendre quadrature method. We have tested the convergence of the results by increasing the number of quadrature points

A. *Electron-helium scattering*

The present method is expected to be valid in the intermediate energy range where FBA fails Calculations have been performed for incident projectile energies from 50 eV to 3000 eV. In this energy range the experimental data are also available

Figure 1 represents the present results for the elastic differential cross section (with and without the inclusion of exchange) at 100 and 300 eV. We have included in the figure the theoretical results obtained by Byron and Joachain (1976) using the optical model formalism along with the recent experimental values of Janson *et al* (1976). At the energies shown in figure 1 our results are in agreement with the measured values at angles $> 10°$ Jansen *et al* however given results upto 50° scattering angle. The agreement between the present results with those of Joachain (1976) is quite satisfactory throughout the angular range except for angles $< 15°$. For electron scattering angles smaller than 10°, a relative comparison with the measurement of Jansen *et al* reveals that our curves underestimate the observed differential cross section while the values predicted by Byron and Joachain over-estimate them. We here hasten to add that our result at 5° for 100 eV electrons is nearly three times greater in magnitude than that given by the FBA.

Table 1 shows our results for differential cross section throughout the whole angular range at electron energies 50, 100, 300, 700, 1000, 2000 and 3000 eV. It is evident from this table that the effect of exchange is appreciable for relatively less energetic electrons as expected. We have however tabulated cross sections with the inclusion of exchange up to 1000 eV only. Above this energy exchange is found to be negligible.

In tables 2, 3 and 4 we compare the present values of the elastic differential cross section for electron energies 200, 400 and 500 eV respectively with various theoretical and experimental results. Of the few theoretical results we include here only those predicted by the static exchange approximation (1973) and the optical theory (1977b). There have however been many recent experimental

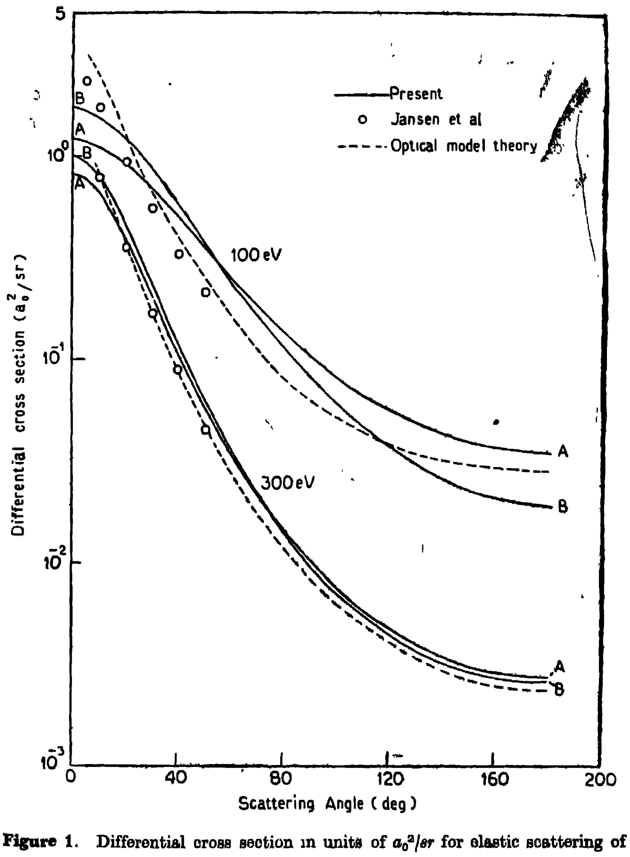

Figure 1. Differential cross section in units of a_0^2/sr for elastic scattering of electrons by atomic helium at incident energies 100 and 300 eV. Solid curve A : present work without exchange effect, solid curve B : present work with exchange effect, broken curve : optical model theory, open circle : experimental points of Jansen *et al.*

Table 1. Differential cross section (a_0^2/sr) for the elastic scattering of electrons by helium in the energy range 50–3000 eV as obtained from the present fixed scatterer approximation. Numbers in parentheses are powers of 10.

θ (deg) \ E(eV)		50	100	300	700	1000	2000	3000
2·5	A	1·81	1·17	7·82(−1)	6·70(−1)	6·40(−1)	5·89(−1)	5·57(−1)
	B	2·27	1·64	9·67(−1)	7·44(−1)	6·90(−1)		
5·0	A	1·80	1·16	7·52(−1)	5·11(−1)	5·62(−1)	4·60(−1)	3·90(−1)
	B	2·24	1·62	9·27(−1)	6·78(−1)	6·05(−1)		
7·5	A	1·78	1·13	7·05(−1)	5·29(−1)	4·60(−1)	3·21(−1)	2·39(−1)
	B	2·19	1·58	8·66(−1)	4·85(−1)	4·94(−1)		
10·0	A	1·75	1·10	6·46(−1)	4·38(−1)	3·57(−1)	2·10(−1)	1·39(−1)
	B	2·13	1·52	7·90(−1)	4·83(−1)	3·82(−1)		
15·0	A	1·68	1·01	5·12(−1)	2·78(−1)	1·97(−1)	8·65(−2)	4·80(−2)
	B	2·02	1·38	6·02(−1)	3·01(−1)	2·09(−1)		
20·0	A	1·59	9·05(−1)	3·85(−1)	1·65(−1)	1·06(−1)	3·78(−2)	1·90(−2)
	B	1·85	1·21	4·58(−1)	1·78(−1)	1·11(−1)		
25·0	A	1·48	7·93(−1)	2·80(−1)	9·85(−2)	5·80(−2)	1·82(−2)	8·66(−3)
	B	1·65	1·02	3·28(−1)	1·05(−1)	6·07(−2)		
30·0	A	1·36	6·82(−1)	2·01(−1)	6·02(−2)	3·34(−2)	9·60(−3)	4·43(−3)
	B	1·46	8·53(−1)	2·31(−1)	6·37(−2)	3·46(−2)		
35·0	A	1·25	5·80(−1)	1·45(−1)	3·80(−2)	2·02(−2)	5·50(−3)	2·49(−3)
	B	1·26	6·98(−1)	1·63(−1)	3·99(−2)	2·08(−2)		
40·0	A	1·13	4·89(−1)	1·05(−1)	2·49(−2)	1·29(−2)	3·37(−3)	1·51(−3)
	B	1·06	5·64(−1)	1·16(−1)	2·59(−2)	1·32(−2)		
50·0	A	9·12(−1)	3·45(−1)	5·77(−2)	1·19(−2)	5·91(−3)	1·48(−3)	6·56(−4)
	B	7·60(−1)	3·65(−1)	6·13(−2)	1·21(−2)	5·98(−3)		
60·0	A	7·30(−1)	2·45(−1)	3·39(−2)	6·39(−3)	3·11(−3)	7·64(−4)	3·36(−4)
	B	5·26(−1)	2·39(−1)	3·48(−2)	6·44(−3)	3·12(−3)		
70·0	A	5·86(−1)	1·77(−1)	2·13(−2)	3·79(−3)	1·83(−3)	4·43(−4)	1·95(−4)
	B	3·60(−1)	1·58(−1)	2·13(−2)	3·80(−3)	1·83(−3)		
80·0	A	4·75(−1)	1·32(−1)	1·43(−2)	2·44(−3)	1·17(−3)	2·82(−4)	1·24(−4)
	B	2·51(−2)	1·09(−1)	1·40(−2)	2·43(−3)	1·17(−3)		
90·0	A	3·90(−1)	1·01(−1)	1·01(−2)	1·68(−3)	8·04(−4)	1·93(−4)	8·44(−5)
	B	1·78(−1)	7·79(−2)	9·77(−3)	1·67(−3)	7·99(−4)		
100·0	A	3·26(−1)	7·96(−2)	7·49(−3)	1·23(−3)	5·83(−4)	1·40(−4)	6·13(−5)
	B	1·27(−1)	5·80(−2)	7·21(−3)	1·22(−3)	5·82(−4)		
120·0	A	2·42(−1)	5·43(−2)	4·70(−3)	7·59(−4)	3·58(−4)	8·58(−5)	4·69(−5)
	B	7·14(−2)	3·57(−2)	4·50(−3)	7·54(−4)	3·57(−4)		
140·0	A	1·95(−1)	4·16(−2)	3·44(−3)	5·48(−4)	2·59(−4)	6·19(−5)	2·71(−5)
	B	4·53(−2)	2·53(−2)	3·29(−3)	5·46(−4)	2·58(−4)		
160·0	A	1·71(−1)	3·55(−2)	2·87(−3)	4·54(−4)	2·15(−4)	5·13(−5)	2·25(−5)
	B	3·41(−2)	2·05(−2)	2·75(−3)	4·51(−4)	2·13(−4)		
180·0	A	1·64(−1)	3·37(−2)	2·70(−3)	4·28(−4)	2·02(−4)	4·83(−5)	2·11(−5)
	B	2·72(−2)	1·92(−2)	2·59(−3)	4·27(−4)	2·02(−4)		

A—present results without exchange; B—present results with exchange.

Table 2. Comparison of various theoretical and experimental differential cross sections for elastic electron-helium scattering at an incident-electron energy of 200 eV. All results are in a_0^2/sr. Numbers in parentheses are powers of 10.

θ (deg)	Present theory		Optical model theory	Static plus exchange	Experimental Values					
	A	B			Crooks and Rudd	Bromberg	Sethuraman *et al*	Jansen *et al*	Jost *et al*	Vriens *et al*
2·5	8·77(−1)	1·16								
5 0	8 54(−1)	1·12	1·98	8 22(−1)		1 73		1 68	2·36	2 04
10 0	7·71(−1)	1·00	1 25	7·37(−1)	1·93	1 12		1·08	1 05	1·28
15·0	6 56(−1)	8·44(−1)	8 33(−1)	6·21(−1)				7 39(−1)	9 65(−1)	8·36(−1)
20·0	5·34(−1)	6·76(−1)	5 75(−1)	5·01(−1)	7·13(−1)	5 27(−1)		5·28(−1)	6·07(−1)	5·61(−1)
25·0	4·21(−1)	5·22(−1)	4·06(−1)	3·90(−1)		3 78(−1)		3·85(−1)	4 45(−1)	3·99(−1)
30·0	3 27(−1)	3·95(−1	2 91(−1)	2 99(−1)	3 25(−1)		2·63(−1)	2·81(−1)	3 20(−1)	2·75(−1)
40·0	1·93(−1)	2·21(−1)	1·55(−1)			1 52(−1)	1·57(−1)	1 51(−1)		
50·0	1·16(−1)	1·26(−1)	8·80(−2)	1·03(−1)	1·03(−1)	8 91(−2)	9·30(−2)	8·85(−2)	1·00(−1)	
60·0	7·27(−2)	7·48(−2)	5·43(−2)			5·57(−2)	5·38(−2)			
70·0	4·78(−2)	4·72(−2)	3·57(−2)	4·23(−1)	4·23(−2)	3·72(−2)	3 57(−2)		4·11(−2)	
80·0	3 30(−2)	3·15(−2)	2·49(−2)			2·63(−2)	2 47(−2)			
90·0	2·38(−2)	2·23(−2)	1·84(−2)	2·17(−2)	2·33(−2)	1·90(−2)	1·77(−2)		2·00(−2)	
110·0	1·41(−2)	1·28(−2)	1·14(−2)	1·34(−2)	1·41(−2)	1·18(−2)	1·15(−2)		1 21(−2)	
130·0	9·77(−3)	8·73(−3)	8·20(−3)	9·64(−3)	1·05(−2)		8·00(−3)		8·57(−3)	
150·0	7·70(−3)	6·82(−3)	6·65(−3)	7·81(−3)	8·43(−3)		6·00(−3)			
170·0	6·86(−3)	5·96(−3)	6 01(−3)							
180·0	6·76(−3)	5·89(−3)	5·94(−3)	6·97(−3)						

A—present results without exchange effect
B—present results with exchange effect.

Table 3. Comparison of various theoretical and experimental differential cross sections for elastic electron-helium scattering at an incident-electron energy of 400 eV. All results are in a_0^2/sr. Numbers in parentheses are powers of 10.

θ (deg.)	Theoretical Values: Present theory A	Present theory B	Static plus exchange	Experimental Values: Vriens *et al*	Chamberlain *et al*	Crooks and Rudd	Jost *et al*	Jansen *et al*
2·5	7·35(−1)	8·71(−1)						
5·0	6·97(−1)	8·24(−1)	6·86(−1)	1·15	1·04		1·39	1·06
10·0	5·71(−1)	6·70(−1)	5·57(−1)	6·88(−1)	6·22(−1)	7·61(−1)	8·93(−1)	6·44(−1)
15·0	4·24(−1)	4·92(−1)	4·11(−1)	4·20(−1)	3·80(−1)		5·82(−1)	
20·0	2·97(−1)	3·41(−1)	2·85(−1)	2·62(−1)	2·37(−1)	3·17(−1)	3·64(−1)	2·89(−1)
25·0	2·30(−1)	2·29(−1)	1·93(−1)	1·65(−1)	1·49(−1)		2·35(−1)	
30·0	1·38(−1)	1·53(−1)	1·30(−1)	1·05(−1)	9·48(−2)	1·41(−1)	1·55(−1)	1·28(−1)
50·0	3·43(−2)	3·58(−2)	3·22(−2)			3·34(−2)	3·57(−2)	3·06(−2)
70·0	1·19(−2)	1·19(−2)	1·14(−2)			1·17(−2)	1·21(−2)	
90·0	5·47(−3)	5·38(−3)	5·34(−3)			6·60(−3)	5·18(−3)	
110·0	3·11(−3)	3·05(−3)	3·10(−3)			3·33(−3)	3·00(−3)	
130·0	2·10(−3)	2·06(−3)	2·12(−3)			2·32(−3)	1·96(−3)	
150·0	1·64(−3)	1·62(−3)	1·67(−3)			1·83(−3)		
180·0	1·43(−3)	1·41(−3)	1·34(−3)					

A—present results without exchange effect
B—present results with exchange effect.

observations for the electron-helium atom system and we insert here the values of Crooks and Rudd (1971), Sethuraman *et al* (1974), Jansen *et al* (1976), Jost *et al* (1973), Vriens *et al* (1968), Chamberlain *et al* (1970) and Oda *et al* (1972). The present values without the inclusion of exchange compare well with those yielded by the static exchange approximation throughout the whole angular range. The effect of exchange enhances the cross section appreciably in the forward direction for the energies considered. These values are in good accord with the results of Byron and Joachain (1977b). When comparisons are made with the different experimental observations, we find that our values of the differential cross section are also in good agreement with all the observed results. In particular for angles $\geqslant 10°$, our results compare nicely with those of Crooks and Rudd (1971), Sethuraman *et al* (1974) and Jose *et al* (1973) at all energies. In addition we also note from Table 4 that for 400 eV electrons, the present set of values with exchange effect is in close agreement with the observed data of

Table 4. Comparison of various theoretical and experimental differential cross sections for elastic electron-helium scattering at an incident-electron energy of 500 eV. All results are in a_0^2/sr. Numbers in parentheses are power of 10.

θ (deg)	Present theory		Optical model theory	Experimental values			
				Jansen *et al*	Bromberg	Oda *et al*	Sethu-raman
2·5	7·06(−1)	8·12(−1)					
5·0	6·61(−1)	7·59(−1)	9·34(−1)	9·06(−1)	9·47(−1)		
10·0	5·17(−1)	5·90(−1)	5·63(−1)	5·50(−1)	5·71(−1)	5·87(−1)	
15·0	3·62(−1)	4·09(−1)	3·56(−1)	3·55(−1)		3·79(−1)	
20·0	2·39(−1)	2·67(−1)	2·23(−1)	2·29(−1)	2·26(−1)	2·28(−1)	
25·0	1·54(−1)	1·70(−1)	1·40(−1)	1·46(−1)	1·43(−1)	1·41(−1)	
30·0	1·00(−1)	1·01(−1)	8·90(−2)	9·33(−2)	9·15(−2)	9·22(−2)	9·41(−2)
40·0	4·52(−2)	4·79(−2)	3·39(−2)	4·12(−2)	4 10(−2)	4·28(−2)	4·13(−2)
50·0	2·26(−2)	2·34(−2)	1·95(−2)	2 03(−2)	2·02(−2)	2·05(−2)	1·85(−2)
60·0	1·25(−2)	1·27(−2)	1 08(−2)		1·13(−2)	1·16(−2)	1·11(−2)
70·0	7·56(−3)	7·57(−3)	6·58(−3)		6·87(−3)		6 10(−3)
80·0	4·92(−3)	4·89(−3)	4·30(−3)		4·48(−3)		3·83(−3)
90·0	3·41(−3)	3·38(−3)	3 01(−3)		3·13(−3)		2·54(−3)
100·0	2·50(−3)	2·47(−3)	2 22(−3)		2·30(−3)		1 78(−3)
110·0	1·93(−3)	1·90(−3)	1·71(−3)		1·79(−3)		1·31(−3)
120·0	1·55(−3)	1·53(−3)	1·38(−3)				9·30(−4)
130·0	1·30(−3)	1·29(−3)	1·16(−3)				7·60(−4)
140·0	1·12(−3)	1 12(−3)	1·01(−3)				7 30(−4)
150·0	1·00(−3)	1·00(−3)	9·04(−4)				6·70(−4)
160·0	9·34(−4)	9 32(−4)	8·38(−4)				
170·0	8·92(−4)	8·88(−4)	8·01(−4)				
180·0	8·79(−4)	8·74(−4)	7·90(−4)				

A—present results without exchange effect
B—present results with exchange effect.

Bromberg (1974) and Oda *et al* (1972). For small angle scattering of relatively less energetic electrons there lies a mark disagreement between the present theoretical values and the experiment. With increase of energy the agreement however seems to be better. The discrepancy in the forward direction may be due to the fact that we hav .not taken into account the effect of atomic distort tion. It is imbedded in the fixed scatterer model itself The inclusion of higher-order terms will definitely improve upon the present situation, particularly in the low energy region. The angular distribution of cross section may then be expected to yield more encouraging feature.

In Table 5, we showed the total elastic cross section values along with the theoretical results by the FBA, the first Born-Oppenheimer approximation, optical model theory (1977b), EBS approximation and also the experimental points of de Heer and Jansen (1975). Both the present two sets of total cross section values are higher in magnitude than that given by FBA and Born-Oppenheimer approximation throughout the energy range. Our values agrees quite satisfactorily with the observed data and de Heer (1975). The cross section yielded by the optical model theory and EBS approximation also are in good accord with our computed results. Recently Dewangan and Walters (1977), have computed total cross sections for scattering of electrons and positrons by atomic helium in the distorted wave second Born approximation. Below 500 eV, our values are larger than theirs (not shown in the table), but above this energy the two sets of results agree very closely.

Table. 5. Total elastic cross section (in units of a_0^2) for electron scattering by helium. Numbers in parentheses are powers of 10.

Energy (eV)	Frist Born Approx.	Born-Oppen-heimer Approx.	Present theory A	Present theory B	Optical model theory	EBS theory	de Heer and Jansen 1975
50	2·20	1 11	6·89	5·08			
100	1·28	1·17	2·56	2 74	2·52	2·41	2·12
200	6·93(−1)	7·55(−1)	1·02	1·18	1·04	1·04	9·77(−1)
300	4·74(−1)	5·23(−1)	6·21(−1)	7·05(−1)	6·23(−1)	6·36(−1)	6·05(−1)
400	3·60(−1)	3·98(−1)	4·43(−1)	4·94(−1)	4·43(−1)	4·53(−1)	4·55(−1)
500	2·91(−1)	3·15(−1)	3·44(−1)	3·77(−1)	3·43(−1)	3·51(−1)	3 48(−1)
700	2·09(−1)	2·23(−1)	2·36(−1)	2·54(−1)	2·35(−1)	2·41(−1)	2·32(−1)
1000	1·48(−1)	1·55(−1)	1·61(−1)	1 70(−1)			1·55(−1)
2000	7·44(−2)		7·77(−2)				7·40(−2)
3000	4·98(−2)		5·12(−2)				4·78(−2)

A—present results without exchange effect

B—present results with exchange effect.

B *Positron-helium scattering*

The study of positron scattering by the helium atom attracted much attention from the theorists, since there has been a number of recent measurements on the total cross section for the system. As we have in this work computed cross sections for only the elastic scattering of positron, we satisfy ourselves in comparing them with other theoretical predictions alone.

Table 6 shows our results for the differential cross section in the full angular range for incident positron energies ranging from 100 to 3000 eV. The present values at all the energies shows fall monotonically from a forward peak and do not seem to show either a minimum or a zero. For relatively less energetic positrons and small scattering angles, these values compare well with the EBS results of Byron and Joachain (1977a). But at higher values of scattering angles

Table 6. Differential cross-section (in a_0^2/sr) for the elastic scattering of Positrons by helium in the energy range 100–3000 eV, as obtained from the present scatterer approximation theory. Number in parentheses are powers of 10.

θ (deg)	Energy (eV) 100	200	300	400	500	700	1000	2000	3000
2·5	2·37(−1)	3 95(−1)	4·59(−1)	4·92(−1)	5·12(−1)	5·32(−1)	5·44(−1)	5·43(−1)	5·27(−1)
5·0	2·33(−1)	3·85(−1)	4·41(−1)	4·66(−1)	4·78(−1)	4·85(−1)	4·78(−1)	4·23(−1)	3·69(−1)
7·5	2·28(−1)	3 68(−1)	4·13(−1)	4·27(−1)	4·30(−1)	4·19(−1)	3·91(−1)	2·95(−1)	2·25(−1)
10·0	2·20(−1)	3·46(−1)	3 77(−1)	3·80(−1)	3·73(−1)	3·46(−1)	3·03(−1)	1·93(−1)	1·31(−1)
15·0	2·01(−1)	2·92(−1)	2·97(−1)	2·81(−1)	2·59(−1)	2·17(−1)	1·66(−1)	7·91(−2)	4·52(−2)
20·0	1·79(−1)	2·36(−1)	2·21(−1)	1·95(−1)	1·70(−1)	1 29(−1)	8·84(−2)	3·44(−2)	1·78(−2)
25·0	1·54(−1)	1·84(−1)	1·59(−1)	1·32(−1)	1·09(−1)	7·62(−2)	4·83(−2)	1·65(−2)	8·11(−3)
30·0	1·30(−1)	1·41(−1)	1·13(−1)	8·86(−2)	7·02(−2)	4·62(−2)	2·77(−2)	8·71(−3)	4·15(−3)
35·0	1·08(−1)	1·06(−1)	8 04(−2)	6·02(−2)	4·61(−2)	2·91(−2)	1·67(−2)	4·98(−3)	2·33(−3)
40·0	8·94(−2)	8·06(−2)	5·76(−2)	4·17(−2)	3·11(−2)	1·90(−2)	1·06(−2)	3·05(−3)	1·41(−3)
50·0	6·02(−2)	4·70(−2)	3·10(−2)	2·13(−2)	1·54(−2)	8·98(−3)	4·85(−3)	1·34(−3)	6·15(−4)
60·0	4·07(−2)	2·86(−2)	1 79(−2)	1·19(−2)	8·45(−3)	4·82(−3)	2·55(−3)	6 93(−4)	3·15(−4)
70·0	2·81(−2)	1·84(−2)	1·11(−2)	7 28(−3)	5 08(−3)	2·86(−3)	1·50(−3)	4·02(−4)	1·83(−4)
80·0	2·00(−2)	1·25(−2)	7·38(−3)	4 77(−3)	3·30(−3)	1·84(−3)	9·59(−4)	2·56(−4)	1·16(−4)
90·0	1·47(−2)	8.90(−3)	5 19(−3)	3·32(−3)	2·29(−3)	1·27(−3)	6·59(−4)	1·75(−4)	7·93(−5)
120·0	7·24(−3)	4·21(−3)	2 40(−3)	1·52(−3)	1·04(−3)	5·71(−4)	2·95(−4)	7·80(−5)	3·53(−5)
150·0	4·83(−3)	2·78(−3)	1·58(−3)	9·93(−4)	8·71(−4)	3·71(−4)	1·91(−4)	5·04(−5)	2·27(−5)
180·0	4·23(−3)	2·44(−3)	1·38(−3)	8·67(−4)	5·91(−4)	3·23(−4)	1·66(−4)	4·39(−5)	1·98(−5)

our results for the differential cross section overestimates those predicted by Byron and Joachain. As the projectile energy is increased the agreement is however better.

In figure 2, we have plotted, for visual display, our values of the differential cross section for projectile energies 100 and 400 eV only. In this graph, we have included for comparison the results obtained by Byron and Joachain by using EBS approximation and the optical model theory (1977b). It is evident

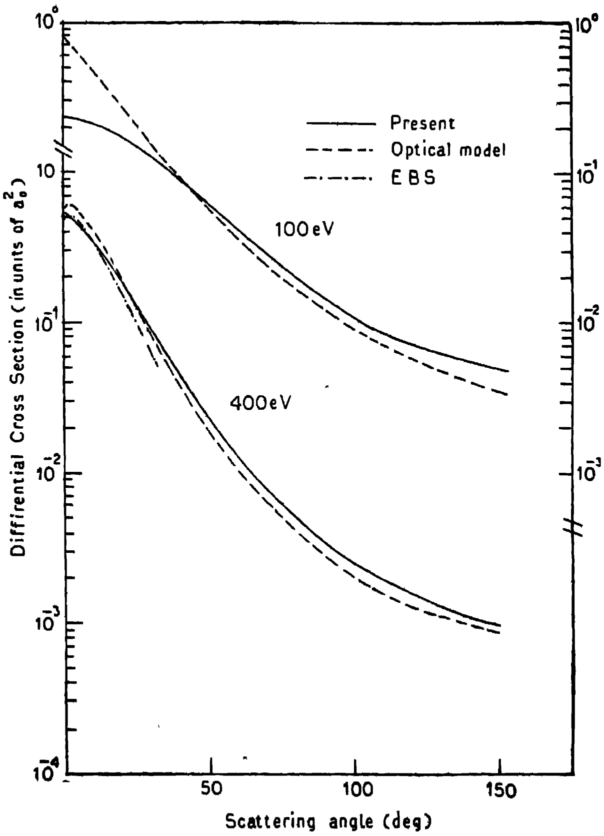

Figure 2. Differential cross section in units of a_0^2/sr for elastic scattering vf positrons by atomic helium at incident energies 100 and 400 eV. Solid curve : present work, broken curve : optical model calculation, dotted chain ; eikonal-Born series approximation.

from the figure that while at small angles of scattering the present values of the differential cross section are lower, at higher angles they overestimate both of the other two sets of results

Table 7 displays our results of the total elastic cross section along with FBA values. In this table we also include the theoretical predictions from Byron and Joachain (1977a and 1977b). The present results agree nicely with the optical model results throughout the whole energy range. The EBS results on the other hand are higher than the present values for low projectile energies. The FBA total elastic cross sections which are same for electrons and positrons are still higher. But with the increase of energy the difference between these values tend to diminish. The computed cross sections of Dewangan and Walters (1977) by using the distorted-wave second Born approximation (not shown in the table) overestimate our values at relatively lower energies. For positron energies above 300 eV these two sets of results agree however very closely. It may be mentioned that the total cross sections for the elastic scattering of positrons by the pure static field of the atom as reported by Dewangan (1977) overestimate all the above mentioned results for energies upto 1000 eV.

Table 7. Total elastic cross sections (in units of a_0^2) for positron scattering by helium. Numbers in parentheses are powers of 10.

Energy (eV)	First Born Approx.	Present theory	Optical model theory	EBS theory
50	2·20	1·30(−1)		
100	1·28	4 45(−1)	5 39(−1)	1·15
200	6·93(−1)	4·28(−1)	4·11(−1)	5·07(−1)
300	4·74(−1)	3·47(−1)	3·27(−1)	3·61(−1)
400	3·60(−1)	2·86(−1)	2·71(−1)	2·88(−1)
500	2·91(−1)	2·41(−1)	2·30(−1)	2·40(−1)
700	2·09(−1)	1·84(−1)		
1000	1·48(−1)	1·35(−1)		
2000	7·44(−2)	7·11(−2)		
3000	4·98(−2)	4·83(−2)		

4. Concluding Remarks

In the present work we have applied the fixed scattered approximation as proposed by Ghosh (1977). This method is very simple and is based on sound theoretical basis. The FSA is able to predict quite reliable results for intermediate and high energies of impact It is able to distinguish between the cases of electrons as projectile and positrons as projectiles. The computational labour involved for the direct process is comparable to the FBA. For the evaluation of the exchange amplitude, one has to perform one dimensional integration. As has been shown by Ghosh (1977) previously for the electron-hydrogen atom scattering and also as we have been able to shown in the present investigation for the elastic scattering of electrons and positrons by atomic helium, this simple method is able to yield results that have been obtained by more sophisticated methods.

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Appendix I

The surviving expression for the double scattering term T^d in the direct channel may be written as

$$T^d = \frac{-4}{\pi^2} \int \frac{\Phi_i^2(r_2, r_3)[2^{i(\boldsymbol{k}''-\boldsymbol{k}_f).\boldsymbol{r}_2} + 2e^{i(\boldsymbol{k}_i-\boldsymbol{k}'').r_3} + e^{i(\boldsymbol{k}''-\boldsymbol{k}_f).r_2}\, e^{i(\boldsymbol{k}_i-\boldsymbol{k}'').r_3}]}{(\boldsymbol{k}''-\boldsymbol{k}_i)^2(\boldsymbol{k}''-\boldsymbol{k}_f)^2(k''^2-k_i^2-i\epsilon)} \times d\boldsymbol{k}''d\boldsymbol{r}_2\, d\boldsymbol{r}_3.$$

The contribution from the first and second term of the parentheses are identical. Thus we required to calculate the expressions D_1 and D_2 only where

$$D_1 = \frac{-8}{\pi^2} \int \frac{\phi_i^2(\boldsymbol{r}_2, \boldsymbol{r}_3) e^{i(\boldsymbol{k}''-\boldsymbol{k}_f).\boldsymbol{r}_3} d\boldsymbol{k}''\, d\boldsymbol{r}_2\, d\boldsymbol{r}_3}{(\boldsymbol{k}''-\boldsymbol{k}_i)^2(\boldsymbol{k}''-\boldsymbol{k}_f)^2(k''^2-k_i^2-i\epsilon)}$$

$$= \frac{-32N^2}{k_i} \sum_{j=1}^{3} a_j \frac{\partial}{\partial\lambda_j} \left[\frac{1}{\lambda_j^2} \frac{1}{k^2+\lambda_j^2} \tan^{-1} \frac{\lambda_j}{2k_i} \right]$$

where

$$a_{1,2,3} = 1, 2c, c^2,$$

$$1, 2, 3 = 2z, 3z, 4z$$

and

$$K = K_i - K_f$$

$$D_2 = \frac{-4}{\pi^2} \int \frac{\phi_i^2(\boldsymbol{r}_2, \boldsymbol{r}_3)\, e^{i(\boldsymbol{k}''-\boldsymbol{k}_f)}\, e^{i(\boldsymbol{k}_i-\boldsymbol{k}'').\boldsymbol{r}_3}}{(\boldsymbol{k}''-\boldsymbol{k}_i)^2(\boldsymbol{k}''-\boldsymbol{k}_f)^2(k''^2-k_i^2-i\epsilon)}\, d\boldsymbol{k}''\, d\boldsymbol{r}_2\, d\boldsymbol{r}_3$$

$$= -64N^4 \sum_{i=1}^{3} \sum_{j=1}^{3} A_i A_j \frac{\partial}{\partial\lambda_i} \frac{\partial}{\partial\lambda_i} \left[\frac{1}{\lambda_i^2\lambda_j^2} \left\{ \frac{\tan^{-1} \frac{\lambda_j}{2k_i}}{k_i(K^2+\lambda_j^2)} + \frac{\tan^{-1} \frac{\lambda_i}{2k_i}}{k_i(k^2+\lambda_i^2)} - \frac{1}{M}\tan^{-1} \frac{2AM}{A^2+B^2-M^2} \right\} \right]$$

where

$$
\begin{aligned}
k &= k_i - k_f \\
M &= [2K_i^2K^2(\lambda_i^2+\lambda_j^2)+K^2\{K_i^2K^2+\lambda_i^2\lambda_j^2+K_i^2(\lambda_i^2-\lambda_j^2)\}]^{\frac{1}{2}} \\
A &= \lambda_i\lambda_j(\lambda_i+\lambda_j) \\
B &= -K_i[K^2+(\lambda_i+\lambda_j)^2] \\
A_i &= a_i \\
A_{1,2,3} &= 1,\ 2c,\ c^2.
\end{aligned}
$$

Appendix II

$$
T^g = \frac{-1}{2\pi^3} \int \phi_f(\boldsymbol{r}_2, \boldsymbol{r}_3)\, e^{-i\boldsymbol{k}_f.\boldsymbol{r}_2} \left(-\frac{2}{r_2}+\frac{1}{r_{12}}+\frac{1}{r_{23}}\right)\phi_i(\boldsymbol{r}_2, \boldsymbol{r}_3)\, e^{i\boldsymbol{k}''.\boldsymbol{r}_1}
$$

$$
\times \frac{2-e^{i(\boldsymbol{k}_i-\boldsymbol{k}'').\boldsymbol{r}_2}-e^{i(\boldsymbol{k}_i-\boldsymbol{k}'').\boldsymbol{r}_3}}{(\boldsymbol{k}''-\boldsymbol{k}_i)^2(k''^2-k_i^2-i\epsilon)}\, d\boldsymbol{k}''d\boldsymbol{r}_1\, d\boldsymbol{r}_2\, d\boldsymbol{r}_3
$$

$$
= I_1+I_2+I_3+I_4+I_5+I_6+I_7+I_8+I_9.
$$

The expressions for I_i's are very lengthy. We only show three type integrals Other integrals can be evaluated in similar manner. In the worst case we require to perform a one-dimensional integral numerically

$$
A^1{}_1 = \int e^{-\lambda_1 r_1}\, e^{-\lambda_2 r_2}\, e^{-\lambda_3 r_3}\, \frac{e^{-i\boldsymbol{k}_f.\boldsymbol{r}_2}}{r_2}\, \frac{e^{i\boldsymbol{k}''.\boldsymbol{r}_2}\, d\boldsymbol{r}_1 d\boldsymbol{r}_2\, d\boldsymbol{r}_3\, d\boldsymbol{k}''}{(\boldsymbol{k}''-\boldsymbol{k}_i)^2(k''^2-k^2-ic)}
$$

$$
= \frac{-128\pi^3}{\lambda_3^3(K_f^2+\lambda_2^2)}\, \frac{\partial}{\partial\lambda_1} \int \frac{d\boldsymbol{k}''}{(K''^2-K_i^2-i\epsilon)(\boldsymbol{k}''-\boldsymbol{k}_i)^2(K''^2+\lambda_1^2)}
$$

$$
= \frac{-128\pi^5}{K_i\lambda_3^3(K_f^2+\lambda_2^2)}\, \frac{\partial}{\partial\lambda_1} \left[\frac{\tan^{-1}\left(\frac{\lambda_1^2-k_i^2}{2k_1\lambda_1}\right)}{K_i^2+\lambda_1^2}\right]
$$

$$
A_2^1 = \int e^{-\lambda_1 r_1}\, e^{-\lambda_2 r_2}\, e^{-\lambda_3 r_3}\, \frac{e^{-i\boldsymbol{k}_f.\boldsymbol{r}_2}}{r_{12}}\, \frac{e^{i\boldsymbol{k}''.\boldsymbol{r}_1}\, d\boldsymbol{r}_1\, d\boldsymbol{r}_2\, d\boldsymbol{r}_3\, dK''}{(K''^2-K_i^2-i\epsilon)(\mathbf{K}''-\mathbf{K}_i)^2}
$$

$$
= \int e^{-\lambda_3 r_3}\, d\boldsymbol{r}_3 \int e^{-\lambda_3 r_3}\, e^{-\lambda_2 r_2}\, \frac{e^{i\boldsymbol{k}_f.\boldsymbol{r}_2}}{r_{12}}\, \frac{e^{i\boldsymbol{k}''.\boldsymbol{r}_1}\, d\boldsymbol{r}_1\, d\boldsymbol{r}_2\, d\boldsymbol{r}_3\, d\boldsymbol{k}''}{(K''^2-K_i^2-i\epsilon)(\mathbf{K}''-\mathbf{K}_i)^2}
$$

$$
= \frac{8\pi}{\lambda_3^3}\, M_1 M_2
$$

where

$$
M_1 = \frac{4\lambda_2}{\pi} \int_0^1 2x\,dx \int \frac{e^{i\boldsymbol{s}.\boldsymbol{r}_1}\, d\boldsymbol{s}}{[(\boldsymbol{S}+\boldsymbol{\Lambda})^2+\mu^2]^3}
$$

$$
= \frac{-4\lambda_2}{\pi} \int_0^1 x\,dx\, \frac{\partial}{\partial\mu^2}\, \frac{\pi^2\, e^{-i\boldsymbol{\Lambda}\cdot\boldsymbol{r}_1}\, e^{-\mu r_1}}{\mu}
$$

and

$$M_1M_2 = -4\lambda_2\pi \int_0^1 xdx \frac{\partial}{\partial\mu^2} \left[\frac{1}{\mu} \int \frac{d\boldsymbol{K}''d\boldsymbol{r}_1\, e^{i\boldsymbol{k}''.\boldsymbol{r}_1}\, e^{-\lambda_1 r_1}\, e^{-i\boldsymbol{\Lambda}.\boldsymbol{r}_1}\, e^{-\mu_1 r_1}}{(K''^2-K_i^2-i\epsilon)(\boldsymbol{K}''-\boldsymbol{K}_i)^2}\right]$$

$$= \frac{16\pi^4\lambda_2}{K_i} \int_0^1 xdx \frac{\partial}{\partial\mu^2} \frac{\partial}{\partial\nu} \left[\frac{1}{\mu} \frac{1}{(\boldsymbol{\Lambda}_i-\boldsymbol{\Lambda})^2+\nu^2} \tan^{-1}\frac{\nu^2+\Lambda^2-K_i^2}{2K_i\nu}\right]$$

where

$$\boldsymbol{\Lambda} = x\boldsymbol{k}_f$$

$$\mu^2 = x\lambda_2^2+x(1-x)k_f^2$$

$$\nu = \lambda_1+\mu.$$

$$A_3' = \int \frac{e^{-\lambda_1 r_1}\, e^{-\lambda_2 r_2}\, e^{-\lambda_3 r_3}\, e^{-i\boldsymbol{k}_f.\boldsymbol{r}_2}\, e^{i\boldsymbol{k}.\boldsymbol{r}_1}\, e^{i(\boldsymbol{k}_i-\boldsymbol{k}'')\boldsymbol{r}_2}}{r_{12}(K''^2-K_i^2-i\epsilon)(\boldsymbol{K}''-\mathbf{K}_i)^2} \times d\boldsymbol{r}_1\, d\boldsymbol{r}_2\, d\boldsymbol{r}_3\, d\boldsymbol{k}''.$$

The above integral has been solved by Ghosh (1977).

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